* a\mid b implies \varphi(a)\mid\varphi(b).
*  n \mid \varphi(a^n-1)       (*a*, *n* > 1)
* \varphi(mn) = \varphi(m)\varphi(n)\cdot\frac{d}{\varphi(d)}       where *d* = gcd(*m*, *n*). Note the special cases
* 
  \varphi(2m) = 
  \begin{cases}
  2\varphi(m) &\text{ if } m \text{ is even} \\
  \varphi(m)   &\text{ if } m \text{ is odd}
  \end{cases}
  

and

* \;\varphi\left(n^m\right) = n^{m-1}\varphi(n).
  
* \varphi(\mathrm{lcm}(m,n))\cdot\varphi(\mathrm{gcd}(m,n)) = \varphi(m)\cdot\varphi(n).

Compare this to the formula       \mathrm{lcm}(m,n)\cdot \mathrm{gcd}(m,n) = m \cdot n.       (See [lcm](https://en.wikipedia.org/wiki/Least_common_multiple)).

* \varphi(n)\; is even for n \geq 3. Moreover, if *n* has *r* distinct odd prime factors, 2^r \mid \varphi(n).
* For any *a* > 1 and *n* > 6 such that  4 \nmid n  there exists an  l \geq 2n  such that  l \mid \varphi(a^n-1).
* \sum_{d \mid n} \frac{\mu^2(d)}{\varphi(d)} = \frac{n}{\varphi(n)}      [[21]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-21)
* \sum_{1\le k\le n \atop (k,n)=1}\!\!k = \frac{1}{2}n\varphi(n)\text{ for }n>1
* \sum_{k=1}^n\varphi(k) = \frac{1}{2}\left(1+ \sum_{k=1}^n \mu(k)\left\lfloor\frac{n}{k}\right\rfloor^2\right)
  =\frac3{\pi^2}n^2+O\left(n(\log n)^{2/3}(\log\log n)^{4/3}\right) ([[22]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-Wal1963-22) cited in [[23]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-23))
* \sum_{k=1}^n\frac{\varphi(k)}{k} = \sum_{k=1}^n\frac{\mu(k)}{k}\left\lfloor\frac{n}{k}\right\rfloor=\frac6{\pi^2}n+O\left((\log n)^{2/3}(\log\log n)^{4/3}\right) [[22]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-Wal1963-22)
* \sum_{k=1}^n\frac{k}{\varphi(k)} = \frac{315\zeta(3)}{2\pi^4}n-\frac{\log n}2+O\left((\log n)^{2/3}\right) [[24]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-24)
* \sum_{k=1}^n\frac{1}{\varphi(k)} = \frac{315\zeta(3)}{2\pi^4}\left(\log n+\gamma-\sum_{p\text{ prime}}\frac{\log p}{p^2-p+1}\right)+O\left(\frac{(\log n)^{2/3}}n\right) [[25]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-25)

(here *γ* is the Euler constant).

* \sum_{1\le k\le n \atop (k,m)=1} 1 = n \frac {\varphi(m)}{m} + 
  O \left ( 2^{\omega(m)} \right ),

where *m* > 1 is a positive integer and ω(*m*) is the number of distinct prime factors of *m*. (*a*, *b*) is a standard abbreviation for gcd(*a*, *b*).[[26]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-26)

**Menon's identity**[[edit](https://en.wikipedia.org/w/index.php?title=Euler%27s_totient_function&action=edit&section=14)]

*Main article:*[*Menon's identity*](https://en.wikipedia.org/wiki/Arithmetic_function#Menon.27s_identity)

In 1965 P. Kesava Menon proved


\sum_{\stackrel{1\le k\le n}{ \gcd(k,n)=1}} \gcd(k-1,n)
=\varphi(n)d(n),


where [*d*(*n*) = σ0(*n*)](https://en.wikipedia.org/wiki/Divisor_function) is the number of divisors of *n*.

**Formulae involving the golden ratio**[[edit](https://en.wikipedia.org/w/index.php?title=Euler%27s_totient_function&action=edit&section=15)]

Schneider[[27]](https://en.wikipedia.org/wiki/Euler%27s_totient_function#cite_note-27) found a pair of identities connecting the totient function, the [golden ratio](https://en.wikipedia.org/wiki/Golden_ratio) and the [Möbius function](https://en.wikipedia.org/wiki/M%C3%B6bius_function) \mu(n). In this section \varphi(n) is the totient function, and 
\phi = \frac{1+\sqrt{5}}{2}= 1.618\dots
 is the golden ratio.

They are:


\phi=-\sum_{k=1}^\infty\frac{\varphi(k)}{k}\log\left(1-\frac{1}{\phi^k}\right)


and


\frac{1}{\phi}=-\sum_{k=1}^\infty\frac{\mu(k)}{k}\log\left(1-\frac{1}{\phi^k}\right).


Subtracting them gives


\sum_{k=1}^\infty\frac{\mu(k)-\varphi(k)}{k}\log\left(1-\frac{1}{\phi^k}\right)=1.


Applying the exponential function to both sides of the preceding identity yields an infinite product formula for [Euler's number](https://en.wikipedia.org/wiki/Euler%27s_number) *e*

e= \prod_{k=1}^{\infty} \left(1-\frac{1}{\phi^k}\right)^\frac{\mu(k)-\varphi(k)}{k}. 

The proof is based on the formulae


\sum_{k=1}^\infty\frac{\varphi(k)}{k}(-\log(1-x^k))=\frac{x}{1-x}
     and     
\sum_{k=1}^\infty\frac{\mu(k)}{k}(-\log(1-x^k))=x, 
     valid for 0 < *x* < 1.

 